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THESIS

COST EFFECTIVENESS OF NAVAL GUNFIRE
A METHODOLOGICAL APPROACH

by

Raymond Guy Zeller

DEC 13 1967

June 1967

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COST EFFECTIVENESS OF NAVAL GUNFIRE
A METHODOLOGICAL APPROACH .

by

⑩

Raymond Guy Zeller

Lieutenant, United States Navy
B.S., University of Illinois, 1960

⑨

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Signature of Author

R. Zeller

Carl R. Jones

Thesis Advisor

Jack R. Boston

Chairman, Department of Operations Analysis

R. F. Rinehart

Academic Dean

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251 450

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ABSTRACT

In this thesis a methodological approach to the determination of the cost effectiveness of naval gunfire support is developed. Two models are presented. The first is a linear program developed by the Ballistics Research Laboratory, in which naval gunfire is employed against a relatively static threat. The second is a probabilistic model wherein the capability of naval gun systems against transient targets is treated. Included is a discussion of the theoretical considerations of the cost of naval gun systems which concludes in the presentation of some alternatives of presenting the results of analysis of this type.

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I. INTRODUCTION

The Problem

This thesis is concerned with developing a cost-effectiveness model for naval gunfire support. It is intended that this model be used to analyze the alternate methods of supplying such support. Historically, naval gunfire support has been almost exclusively associated with amphibious operations. In this historical context the basic task of naval gunfire support units is to support the assault of the objective. The support role can be conveniently ordered into three chronological phases, pre-landing, landing, and post-landing (Ref. 14, p. 1-1).

Or more specifically:

1. Destroying or neutralizing shore installations that oppose the approach of ships and aircraft.
2. Destroying or neutralizing defenses that may oppose the landing.
3. Destroying or neutralizing defenses that may oppose the post-landing advance of the landing force.

Naval gunfire has been extensively employed in Vietnam in situations other than support of amphibious assaults. A few limited amphibious assaults have been conducted in Vietnam; however, none were of the size or complexity of landings in WWII or Inchon in Korea. Despite this fact an increasing demand for support of ground operations by naval units has been experienced (Ref. 9, p. 1). Certain characteristics of naval gunfire and aspects of counterinsurgency warfare have encouraged

its increased use in situations analogous to phase three; that is, the destruction or neutralization of defenses opposing the advance of ground forces. The implication is that naval gunfire is currently being used in roles once reserved for artillery and aircraft.

The problems inherent in coordinating the efforts of ships and aircraft in acknowledged complementary roles in amphibious landings were surmountable. Unfortunately, as the area of concern and the size of the forces expands, the problems of coordination have become increasingly complex. Cost effectiveness provides a possible means of identifying the relative ability of ships and aircraft to perform various tasks. It is hoped that analysis will denote relatively discrete areas wherein ships and aircraft are substitutes; that is, where one of the two systems demonstrates a definite superior performance ability. In the event the systems are complementary, analysis can also be used to distinguish specific individual roles that each system should be assigned in order to maximize the overall effect.

Unfortunately, although considerable cost-effectiveness analysis pertaining to aircraft weapon systems is available, comparable analysis for naval gun systems is practically nonexistent (Ref. 3). Furthermore, although analysis of artillery systems is available, sufficient differences exist in the systems so that the usefulness of such analysis is restricted to that of a guide to technique. For example, a naval gun system consists of a ship and its armament; whereas artillery is peculiar to land armies. In many cases there are differences in the

types of charges and projectiles used by each system. Furthermore, the administrative organizations that support these systems differ vastly.

Thus a revival of interest in naval gunfire disclosed a serious lack of quantitative knowledge of the naval gun systems. For this reason the subject was determined to be a particularly fruitful area of study.

Background of Cost Effectiveness

Economists have long employed the phrase "scarcity of resources". The notion that resources in an economy are scarce is probably not particularly appealing; however, resources are indeed scarce, not in the sense of anthracite coal, perhaps, but scarce in that they are not unlimited. In a present day context, competition between defense and nondefense programs within the federal sector seems to support this contention in a narrower context.

In an attempt to more efficiently utilize our limited resources cost-effectiveness analysis has been introduced in government, particularly the defense department (Ref. 5, p. 1). This procedure constitutes the analytical core of the planning, programming and budgeting method of fiscal control employed in defense expenditures.

Cost-effectiveness analysis can be defined as the systematic analysis of the cost and military effectiveness of alternate methods of accomplishing an objective in the presence of present and future scarce resources. As originally conceived, the role of the decision

maker is not diminished; rather the decision maker supplies the subjective judgments required in assessing nonqualitative aspects and rendering value judgments (Ref. 6, p. 183).

Cost is a factor which must be considered, for it represents expenditure of limited resources. Furthermore, weapon systems are becoming more costly and implications of defense decisions on the economy too vast and far reaching to be disregarded. However, costs and capabilities must be compared. That is, the lowest cost is favorable only in terms of a specified level of effectiveness. For example, in Figure 1 a number of buildings are to be destroyed. Alternatives i ($i = 1, 2, 3, 4$) are available to do the job with costs c_1, c_2, c_3 , and c_4 , respectively ($c_2 = c_4$). Each alternative has the ability to destroy a number of buildings b_i , ($i = 1, 2, 3, 4$, $b_2 = b_3$). Alternative 2 is preferred to alternative 3, as it destroys the same number of buildings for a smaller cost.

An alternate approach may be employed. Suppose, for example, a budget level c_2 has been specified. With a budget specified the object becomes to maximize the effectiveness the number of buildings a system is capable of destroying. Alternative 2 is preferred to alternative 4, which has the same cost, as it (2) destroys more buildings. Now consider alternative 1 and 2. Alternative 2 is more expensive; however, it also destroys more buildings. It might be appropriate to ask if the additional buildings destroyed, $b_2 - b_1$, is worth the additional cost, $c_2 - c_1$. Reference was made to subjective judgments

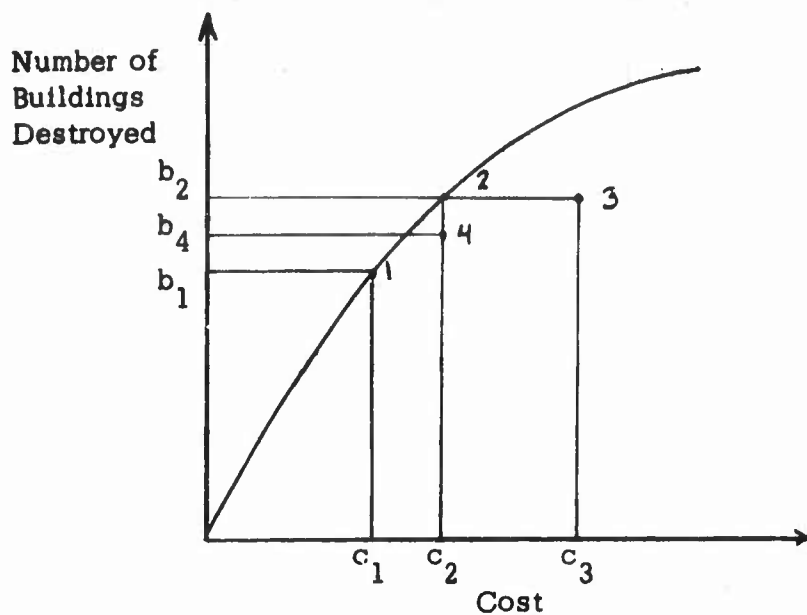


FIGURE 1

and nonqualitative aspects. These notions are best illustrated with an example. Suppose that alternative 4, although destroying less buildings than alternative 2, does so in a much more spectacular manner, thereby instilling fear in our enemies. This fear or loss of morale illustrates the type of effect that is difficult if not impossible to quantify.

Certain advantages are inherent in the techniques of cost-effectiveness analysis. Such analysis will improve the decision making process by employing a systematic presentation of alternatives and outcomes bringing into focus costs and the effect of cost changes on mission effectiveness. In addition, past experience indicates that during the course of the analysis additional alternatives often appear, possibly due to the systematic nature of the analysis and the learning effect experienced (Ref. 2, p. 1).

Distinct disadvantages also exist. Such analysis may omit factors of major importance or treat them improperly. Thus, although a study may have the appearance of scientific objectivity, the results may be severely prejudiced. For example, within programs of the size and complexity of current major weapon systems, elements of cost and effectiveness are frequently difficult to assess. In such cases, studies are often separated into smaller subsets of the original problem which are analytically more tractable. Reference 6 deals extensively with the problems of suboptimization and the opportunities for error which can then arise. Analysts and users should never lose sight of the fact that as the time horizon is extended, the ability to forecast events and circumstances is degraded. In general, a study should make note of any factors with a high degree of uncertainty and examine the sensitivity of the results to variations in such factors (Ref. 7, p. 9).

Chapter II consists of a general discussion of the notion of a scenario and the relevance of the two specific scenarios selected for this study. In Chapter III, two effectiveness models are presented, each of which is applicable to one of two scenarios chosen. Theoretical and practical considerations of the effectiveness of naval gunfire provides motivation for the models chosen. Chapter IV discusses factors that must be taken into account in developing the cost of a weapon system. The results of Chapters III and IV are then married

to provide the cost effectiveness comparison of naval gunfire support,
the central theme of the thesis . The presentation concludes with
recommended areas of further study .

II. SCENARIO

A comparison of the effectiveness of alternative weapon systems is not particularly meaningful unless some frame of reference or environment is prescribed. For example, consider a sprinter who runs a 100-yard dash in the rain and on a muddy track. It would be difficult to compare this sprinter with another who ran the same race on a clear day with perfect conditions. Conversely, if the two sprinters run the same race under the same conditions, the notion that their respective times provides some measure of their comparative abilities is intuitively appealing.

The set of conditions referred to above is called a scenario in cost-effectiveness analysis. In the most formal sense, a scenario is as in the setting of a play wherein the time, location, and other conditions environing and affecting the agents are specified; i.e., the entire set of the essential conditions or of the attendant facts that bear on the subject are specified. Therefore, in order to achieve a basis for comparison of alternative weapon systems, it is necessary to examine the performance (or effectiveness) of competing systems under the same circumstances or within the same scenario.

The use of a scenario to compare effectiveness permits parameterization of the uncertainty inherent in the environment. It is conceivable that weapon systems may experience employment at some time in the future under circumstances which were not considered particularly appropriate at the time of the system's inception. By

constructing various scenarios, it becomes possible to examine performance under a great number of situations.

Within a scenario the assignment of a particular target to be fired on shall be termed a mission. For example, a ship might be assigned a mission to fire on a particular bridge. In the most general sense it is possible to classify missions into two general types, bombardment and call fire. These types will constitute the two scenarios to be used in this analysis.

Bombardment Scenario

The bombardment scenario in this study is defined as consisting of those missions which are of a prearranged or scheduled nature. A specific target is to be neutralized in so far as its military potential is concerned. For example, a bridge is to have a span dropped, buildings are to be destroyed (wall caved in and roof collapsed), roads are to be cratered so as to prevent the movement of men and materials. The benefits of such missions are not expected to have an immediate effect on the enemy's ability to wage war but are more strategic in nature. To further specify the scenario the only friendly forces involved in the bombardment are naval forces. Such missions are expected to occur primarily in such locations as North Vietnam, since guerilla or insurgent forces traditionally avoid fixed positions. A few exceptions might exist in areas where guerillas have enjoyed control for a period of several months that is a sufficient length of time to permit construction of tunnel complexes and staging areas.

Call Fire Scenario

The call fire scenario in this study is defined as those missions fired in support of (at the request of) friendly or allied forces which are in direct contact with enemy units (allied forces are firing on enemy forces). The primary features of this scenario are the lack of fixed positions by either side and the necessity for providing supporting fire as quickly as possible to allied forces. For example, consider the situation where an allied patrol is ambushed by enemy forces. The ambushing forces may have time to dig shallow foxholes for machine guns and similar positions but will not have the advantage of well constructed mortar or artillery positions. Natural cover afforded by features of the terrain will also be available to ambushing forces and will be the only protection to the allied forces which are under attack. Another example of missions falling in this scenario would be those cases in which enemy forces attack the perimeter of an allied base. It has been shown that in such cases as these, where the position of one force is known and the position of the other force is only approximately known, the rate at which ambushing forces inflict casualties on the force that was ambushed is considerably in excess of the rate at which the ambushed force inflicts casualties on the ambushing force (Ref. 4).

Mission Attributes

Within each scenario missions exhibit additional attributes of degree of natural cover, degree of hardness (concrete or dirt), and physical dimensions.

The degree of natural cover may be further subdivided into heavy, moderate, and light. Heavy cover includes such terrain as dense tropical rain forests where heavy undergrowth and large trees are predominant. Moderate cover refers to wooded areas existing in temperate climates, e.g., North America and central Europe. Light cover denotes sparsely wooded areas or areas of infrequent and irregular vegetation including open areas where only grass and shrubs offer cover. Empirical observation has supported the intuitively appealing notion that a dense rain forest will smother the effects of the explosion and fragmentation of projectiles. Regrettably, the quantitative aspects of variations in cover, although under investigation, are not yet known (Ref. 8, p. 8).

The following Figure 2 lists examples of missions and variations of the attributes of hardness and physical dimensions (Ref. 1, p.3).

HARDNESS	SOFT	MEDIUM	HARD
<u>Dimension</u>			
POINT	Artillery (exposed)	Artillery (revetted)	Pill box
LINEAR	Roads Railroads Bridges	Roads Railroads Bridges	Roads Railroads Bridges
AREA	Personnel (offense)	Personnel (prone)	Personnel (defense)

FIGURE 2

Some analysis of the quantitative effect of naval guns on such structures is available from supporting arms evaluation centers; however, it is primarily theoretical in nature and is not well supported by empirical observation or experiments (Ref. 9, p. 11).

Distribution of Missions

Mission distribution denotes the relative frequency with which particular missions, including attributes, occur. For example, in the call fire scenario all missions might occur in either heavy or moderate cover equally divided between each and further equally divided between soft, medium, and hard area targets. Or, using figures to illustrate the foregoing, from a total of 60 missions we could expect 30 each in heavy and moderate cover with 10 soft, 10 medium, and 10 hard targets in heavy and moderate cover, respectively.

For the current conflict in Vietnam there is no necessity for speculation on the initial distribution of missions in the bombardment scenario, for by definition, missions are scheduled and must be known to be scheduled. The location and characteristics or attributes of targets constituting potential missions would be determined from intelligence sources. However, since a study may be conducted several years before a particular conflict occurs, an effort must be made to remain abreast of changing features of areas in which future wars may erupt. Uncertainty will arise since intelligence involves estimating and projecting into the future.

After a war begins and naval gunfire is employed, it is reasonable to expect the distribution of missions to change as the time horizon is extended. It would be difficult, it is true, for major support facilities to be moved or their characteristics significantly altered. Certain facilities are greatly dependent on features of terrain which are stable. An example would be transshipment points where supplies are transferred from ships or barges to trucks for further movement. Such an area would require roads for the trucks and hydrographic conditions suitable for ships to land or transfer supplies. Nevertheless, changes will occur. Where possible, targets would probably be moved further inland out of the range of naval guns. If movement of a target were not feasible, it would most likely be more heavily fortified either offensively (shore batteries) and/or defensively (camouflage or bunkers). Elements which constitute a target would probably be dispersed to create an area target of what had previously been a point or linear target, for example, storing ammunition in smaller quantities and more widely separated positions. All of these actions tend to make a target more difficult to destroy, thus impeding the accomplishment of a mission. Figures 3, 4, and 5 indicate the behavior of mission distribution that may be expected as the time horizon is extended. In this analysis no attempt is made to do more than predict the general direction of the change which might be expected in the distribution of missions.

Trends In Mission Distribution

Percent Missions Occurring as a
Function of Range from the Coast

$$T_1 < T_2 < T_3$$

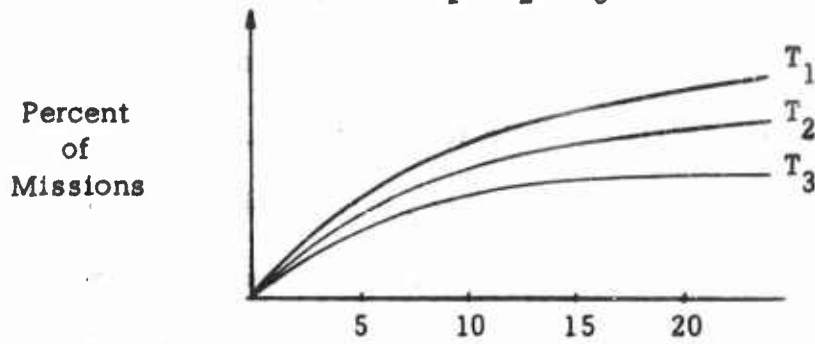


FIGURE 3

Percent Missions Occurring as a
Function of Degree of Hardness

$$T_1 < T_2 < T_3$$

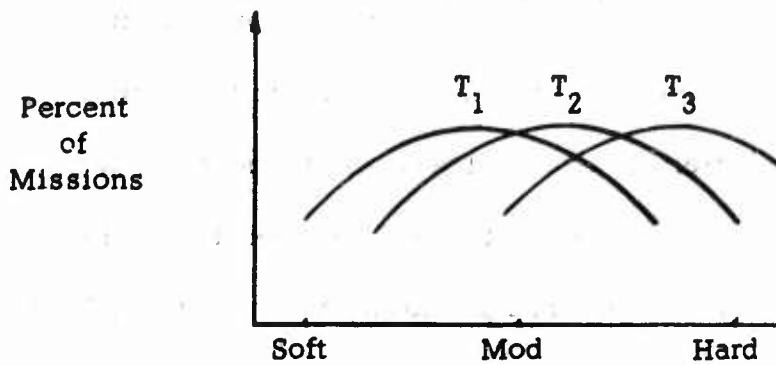


FIGURE 4

Percent Missions Occurring as a
Function of Physical Dimensions

$$T_1 < T_2 < T_3$$

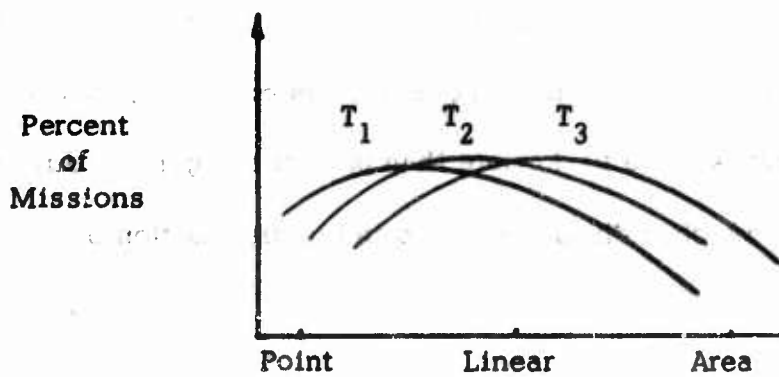


FIGURE 5

The call fire scenario is more limited than the bombardment scenario in that fewer of the potential attributes of missions can occur because of the restrictions imposed on the availability of fixed installations to the enemy and the inherent mobility of insurgent forces. These requirements will, most likely, result in primarily anti-personnel missions with perhaps a few anti-artillery missions. Because of the transient nature of the missions, determining the initial distribution will be somewhat more difficult than in the case of the bombardment scenario. The most recent information on contact with enemy units and accurate intelligence regarding enemy intentions would be required to determine the initial distribution. In the case of a study pertaining to a potential conflict, information regarding the nature of a possible enemy's combat unit organization would be most beneficial, e.g., unit size, type of weapons carried, and tactics.

Since the initial information regarding mission distribution harbors an element of doubt, the ability to predict future distributions is degraded. The problem of predicting the future is lessened somewhat due to the fact that there are fewer mission attributes to worry about. For example, referring to Figure 2, only exposed artillery and personnel in offensive, defensive, or prone postures should be encountered. Where naval gunfire support is employed it would be reasonable to expect enemy units to avoid contact if within range of these guns.

Thus, occurrence of missions as a function of distance from the coast would be expected to exhibit the same behavior as in the case of the bombardment scenario (See Figure 2).

III. EFFECTIVENESS

As was previously mentioned, cost-effectiveness analysis is designed to compare the cost and effectiveness of alternative weapons systems. More specifically, the objective of this analysis is to present in a systematic fashion the cost and effectiveness of alternate choices of weapon systems. In this study a naval gunfire support weapon system is defined as a group or combination of ships assigned to provide such gunfire support. In practice, ships would be assigned to a task unit which is charged with providing supporting fire. Assignments are frequently only temporary; however, for our purposes here they will be assumed to remain constant for the duration of the analysis of a particular group. In other words, substitution of ships of the same type does not affect the analysis; only the task unit composition is significant.

It is the purpose of this chapter to develop a means of determining the ability of a weapon system to produce a desired effect. Desired effect is considered to be the accomplishment of a mission. In a broader sense it could also mean the accomplishment of several missions. Recall that the assignment of a ship to fire on a particular target achieving a specified effect, dropping a span, etc., constitutes a mission.

It was determined that missions could be placed into two general categories, which were then defined as the two scenarios. Within each of the scenarios, accomplishment of a mission has a different

meaning or connotation. In the bombardment scenario, accomplishment of a mission means the destruction of all targets constituting missions falling within this scenario. In the call fire scenario, accomplishment of a mission means a target is brought under fire. For example, a ship is ordered to proceed to a particular target and fire on it (this, of course, constitutes a mission). This mission is considered accomplished as soon as the ship succeeds in placing a round within 100 yards* of the target.

It may happen that several of the alternative weapon systems examined are capable of accomplishing the same mission. This is particularly so in the call fire scenario, for the requirement of placing a round within 100 yards of a target is not particularly stringent. In fact, the only foreseeable impediment to the accomplishment of this mission would be that a target is simply out of range. It will be seen that this does occur in some cases. Thus there must be some means of differentiating between weapon systems which accomplish the same objective, i.e., destroying the same target, and some measure of respective abilities to achieve this desired effect. This measure is usually referred to as a measure of effectiveness. Since the measure of effectiveness is closely related to the accomplishment of the objective, it should not be surprising that the two scenarios have different measures of effectiveness.

*This distance was chosen arbitrarily as representative of the proximity required to force attacking forces to adopt a defensive posture.

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Measures of Effectiveness

In the bombardment scenario the measure of effectiveness is the number of times that a particular weapon system can defeat a specified threat. For example, a ship has available a certain amount of ammunition and is to destroy six bridges. The number of times the ship can defeat the specified threat is that number of times the six bridges could be destroyed with the amount of ammunition the ship has available. The specified threat will be that threat currently in existence and against which alternative weapon systems will be examined to determine which combination of ships can defeat the threat the greatest number of times.

In the call fire scenario the measure of effectiveness chosen is the length of time required for a weapon system to place a round within 100 yards of a target. The necessity for rapid accomplishment of the mission in this scenario has been discussed. Since the emphasis is on rapidity of results, the most effective alternative is that one which accomplishes the mission in the least amount of time.

A measure of effectiveness, though unquestionably valuable, often conceals many of the more esoteric aspects of the situation which is being analyzed. Admittedly it is possible to become so familiar with the practical aspects of a problem that a fresh look is difficult. It is equally true, however, that some appreciation of the practical side of the problem will permit an analyst to recognize

areas where assumptions can simplify the procedure without robbing the analysis of practical value. Practical considerations will be introduced here in a discussion of the characteristics of naval gun systems.

Naval Gun Systems

Naval gun systems consist of both the type of armament (gun) installed on a ship and the ship itself. If discussion is restricted to ships and guns currently in inventory, which it is here, it is not useful to discuss the two separately. For example, in the call fire scenario, the ability of a ship to accomplish a mission depends on whether the gun is of long enough range to reach the target and how fast the ship can move to put the gun within range of the target. Therefore, a naval gun system is defined as a ship and its installed guns.* Several characteristics which distinguish between naval gun systems are available for consideration.

To begin with, some characteristics of naval gun systems are applicable to all systems. Naval gun systems provide a variety of calibers often on the same ship. This permits a more flexible response. For example, it would not be necessary to fire on a junk with a 16-inch gun, since either the 5-inch or the 40-MM gun also installed are capable of destroying a junk, may be operated by fewer personnel, and cost less to fire. In addition there are available

*The same procedure could be applied to hypothetical systems.

different types of projectiles and fuzes adding more flexibility. The initial velocity of naval guns is higher than artillery which results in superior material penetration of the target. Coupled with the higher relative muzzle velocity is a flatter trajectory and a fall of shot pattern which is narrow in deflection (across the line of fire) and long in range (along the line of fire). Because of these factors more accurate fire is possible against targets presenting a face verticle to the line of fire or when firing along the long axis of a target, i.e., a road.

Shipboard fire control systems are sophisticated. Such systems permit fire while the ship is moving and in the case of larger ships, such as cruisers, two or more missions may be conducted simultaneously. In addition, optical and electronic equipment make possible observation of targets when an unobstructed line of sight exists permitting direct fire by the ship. Within the limits imposed by hydrographic conditions, firing ships may be continuously positioned to provide support for units which are themselves mobile. In addition, ships may maneuver to avoid counterbattery or other attack. These points require elaboration since there is evidence indicating that these are to a large extent responsible for the heavy use of naval gunfire in Vietnam. In counterinsurgency warfare the emphasis is on mobility with small unit actions spread throughout the countryside. Conventional artillery has some serious drawbacks in such a situation. Artillery is vulnerable to enemy counteraction (attack)

and time is required to establish communications with spotters and achieve positions from which support may be provided to forward units.

The very existence of artillery in an area may very well cause the enemy to avoid or break contact. Such a result as this is counter to the most basic precepts of counterinsurgency warfare where it is desired to force the generally weaker, numerically, insurgent forces to fight a more or less conventional engagement. Finally, the ability to reposition ships on station permits uninterrupted availability.

Individual gun systems possess factors which may distinguish between systems. Some examples of these factors are ship speed, cruising radius, draft, armament, magazine capability, gun range, and armor. The effect of variations in cruising radius was ignored since ships can be fueled en route or on station, and the speeds of ships that provide the majority of naval gunfire support services are approximately equal (Ref. 9, p. 11). The effect of variations in draft were found to be insignificant for a Vietnam locale (See Appendix A).

The effects of differences in armament are many and varied. Guns of longer range can complete missions beyond the range of guns of lesser range. Furthermore, guns of greater range can provide supporting fire over a greater area of land. Thus long range guns reduce the time required to provide gun fire support since no physical movement of the ship is required. Longer range guns also fire heavier projectiles which have commensurately more lethal effect; that is, in

general it takes less of the heavy projectiles to accomplish the same mission (Ref. 9, p. 11).

Magazine capacity is of particular importance to smaller ships which are frequently limited in their storage capacity. Specifically, greater magazine capacity permits a ship to remain on station providing supporting fire for a longer period of time.

Armor refers to the structural members (plates) of a ship which are designed to deflect enemy shells. Considerable differences exist between ships. For example, battleships are quite heavily armored, whereas destroyers have practically no armor whatsoever. Of course, the presence of armor decreases the vulnerability of ships to counteraction (counterbattery fire or torpedoes). This aspect will not be considered here except for the inclusion of escorts to be considered in the next chapter.

Bombardment Scenario Effectiveness Model

The number of times a weapon system can defeat (destroy) a specific threat was chosen as the measure of effectiveness in the bombardment scenario. One method of determining the number of times the threat can be defeated is to employ a mathematical programming model. Here a linear program developed by the Ballistics Research Laboratory will be used. The model chosen employs the following symbology:

- T_j - denotes the j -th mission class; $j = 1 \dots 81$
- f_j - denotes the number of missions in the j -th class
- w_i - denotes weapon type, $i = 1 \dots 3$
- r_{ij} - denotes the number of rounds required to defeat T_j
with w_i
- n_i - denotes the number of rounds available for w_i
- x_{ij} - denotes the number of targets of the j -th class assigned
to w_i

Consideration in this formulation will be restricted to three types of naval gun systems w_i ; $i = 1, 2$, and 3 with ranges r_i ; $i = 1, 2$, and 3 , respectively. Further $r_1 < r_2 < r_3$.

Referring to the figure outlining mission categories, or classes, it will be seen that there are 27 possible combinations of categories. Since any particular mission can occur in each and every range band $(0, r_1)$, (r_1, r_2) , or (r_2, r_3) , the index j will vary from 1 to 81. T_1 through T_{27} denotes all possible combinations occurring within the $(0, r_1)$ band. These missions may be fired by any of the weapons considered. T_{28} through T_{54} denotes the possible combinations occurring in the range band (r_1, r_2) . These missions may be fired only by weapons two and three (w_2, w_3). Finally, T_{55} through T_{81} denotes missions occurring between ranges (r_2, r_3) . These missions may be fired only by weapon three (w_3).

The number of rounds (n_i) available for w_i will be based on the number of naval gun systems which constitute the weapon system and

the total number of rounds contained in all the ships that constitute the weapon system that can be used by w_1 . For example, consider a weapon system composed of a cruiser and a destroyer. The destroyer carries w_1 and the cruiser w_1 and w_2 . In this case, n_1 will be the number of rounds of ammunition for w_1 carried by both the destroyer and the cruiser. The number of rounds n_2 for use by w_2 are those available in the cruiser.

Relatively strong assumptions are employed in defining the number of rounds (r_{ij}) required to defeat T_j with w_i to facilitate the solution of the linear program. In actual fact, r_{ij} is an increasing function of range. Since range bands have been employed, r_{ij} will be assumed constant for a particular range band. The constant value used will be that which exists at the midpoint of the band, that is, the average for the band. For example, $r_{11} = r_{11}$ at range $\frac{r_1}{2}$ or $r_{1,28} = r_{1,28}$ at range $\frac{r_2 - r_1}{2}$. In the event T_j is out of range of w_i ; $r_{ij} = M$, where M is an arbitrary large positive number. Some mention should be made of the fact that the pattern of the fall of shot at the target is probabilistic and usually described by the bivariate normal distribution (Ref. 8, p. 198). The impact of this behavior is a variation in the number of rounds required to defeat a threat. The mean value was chosen to represent r_{ij} . That is, $r_{1,28} = \bar{r}_{1,28}$ at range $\frac{r_2 - r_1}{2}$. An alternate procedure might be to determine the number of rounds (R) so that a subjective confidence statement of

95% could be made that no more than R rounds will be required to destroy the target. For example, assume r_{ij} at range $\frac{r_2 - r_1}{2}$ is distributed according to $f(r)$ and that $P(r_{ij} \leq R) = \int_0^R f(r) dr = .95^*$. Use $R = r_{ij}$ as a deterministic estimate of r_{ij} . Thus there exists a subjective confidence of 95% that no more than R rounds will be required to defeat T_j .

The formulation of the linear program follows:

Determine the x_{ij} which maximizes K , subject to:

$$\begin{aligned} K[r_{11}x_{11} + r_{12}x_{12} + \dots + r_{1,s}x_{1,s}] &\leq n_1 \\ &\vdots \\ K[r_{m1}x_{m1} + r_{m2}x_{m2} + \dots + r_{ms}x_{ms}] &\leq n_m \\ x_{11} + x_{21} + \dots + x_{m1} &= f_1 \\ &\vdots \\ x_{1,s} + x_{2,s} + \dots + x_{m,s} &= f_s \\ x_{ij} &\geq 0 \end{aligned} \quad (1)$$

The values of K and x_{ij} are not to be restricted to integers. By introducing the slack variables L_i , $i = 1, \dots, m$, we obtain the following linear equations:

*The 95% significance level was chosen arbitrarily.

$$K[r_{11}x_{11} + r_{12}x_{12} + \dots + r_{1s}x_{1s} + L_1] = n_1 \quad (2)$$

$$K[r_{m1}x_{m1} + r_{m2}x_{m2} + \dots + r_{ms}x_{ms} + L_m] = n_m$$

$$x_{11} + x_{21} + \dots + x_{m1} = f_1$$

$$\vdots$$

$$x_{1s} + x_{2s} + \dots + x_{ms} = f_s$$

$$L_1 \geq 0$$

$$\text{By letting } K = \frac{n_1}{[r_{11}x_{11} + r_{12}x_{12} + \dots + r_{1s}x_{1s} + L_1]} \quad (3)$$

the problem becomes

$$\min \frac{1}{n_1} [r_{11}x_{11} + r_{12}x_{12} + \dots + r_{1s}x_{1s} + L_1] \quad (4)$$

subject to:

$$\begin{aligned} n_1[r_{21}x_{21} + r_{22}x_{22} + \dots + r_{2s}x_{2s} + L_2] &= \\ n_2[r_{11}x_{11} + r_{12}x_{12} + \dots + r_{1s}x_{1s} + L_1] & \\ \vdots & \\ \vdots & \end{aligned} \quad (5)$$

$$\begin{aligned} n_1[r_{m1}x_{m1} + r_{m2}x_{m2} + \dots + r_{ms}x_{ms} + L_m] &= \\ n_m[r_{11}x_{11} + r_{12}x_{12} + \dots + r_{1s}x_{1s} + L_1] & \end{aligned}$$

$$x_{11} + x_{21} + \dots + x_{m1} = f_1$$

$$\vdots$$

$$x_{1s} + x_{2s} + \dots + x_{ms} = f_s$$

which is equivalent to

$$(5) \quad 1/n_1 \min[r_{11}x_{11} + r_{12}x_{12} + \dots + r_{1s}x_{1s} + L_1] \quad (6)$$

subject to $AX = b$

$$x_{ij} \geq 0$$

where

$$X = \begin{pmatrix} x_{11} \\ x_{12} \\ \vdots \\ x_{1s} \\ L_1 \\ x_{21} \\ x_{22} \\ \vdots \\ x_{2s} \\ L_2 \\ \vdots \\ x_{m1} \\ x_{m2} \\ \vdots \\ x_{ms} \\ L_m \end{pmatrix}$$

$$b = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ f_1 \\ f_2 \\ \vdots \\ f_s \end{pmatrix} \quad \left. \begin{matrix} \vdots \\ \vdots \\ \vdots \end{matrix} \right\} \text{first } m-1 \text{ terms}$$

$A = (a_{ij})$ is the coefficient matrix.

Any standard linear programming algorithm can be used to solve this problem.

Call Fire Scenario Effectiveness Model

Recall that in the call fire scenario the measure of effectiveness is the time required to take a target under fire, or, more specifically, the object is to fire on a target as quickly as possible. It is assumed that missions are distributed according to $f_{XY}(x,y)^*$ over the area shown in Figure 6a. The approach will be to minimize a function $g(X,Y,a_1,b_1,c_1)$ representing the time required to take a target under fire when that target occurs at position X,Y and where a_1, b_1 , and c_1 represent stationing positions of destroyers, cruisers, and battleships, respectively. The problem may then be represented as:

$$\min_{a_1, b_1, c_1} E[g(X,Y,a_1,b_1,c_1)]^{**} = \int_0^L \int_0^{r_3} g(X,Y,a_1,b_1,c_1) f_{X,Y}(x,y) dx dy$$

$g(X,Y,a_1,b_1,c_1)$ can also be represented as

$$g(X,Y,a_1,b_1,c_1) = \begin{cases} g_1(X,Y,a_1,b_1,c_1) & 0 \leq Y \leq r_1 \\ g_2(X,Y,b_1,c_1) & r_1 \leq Y \leq r_2 \\ g_3(X,Y,c_1) & r_2 \leq Y \leq r_3 \end{cases}$$

The nature of the function $g(X,Y,a_1,b_1,c_1)$ must be determined.

In the figure the time for a ship stationed at point a to move to a position from which a target occurring at X,Y may be fired on is equal

*Reference 12, p. 193-197 discusses the theory and notation of a joint random phenomena (the occurrence of a target in this context).

**Reference 12, p. 203 discusses the notion of expectation as herein used.

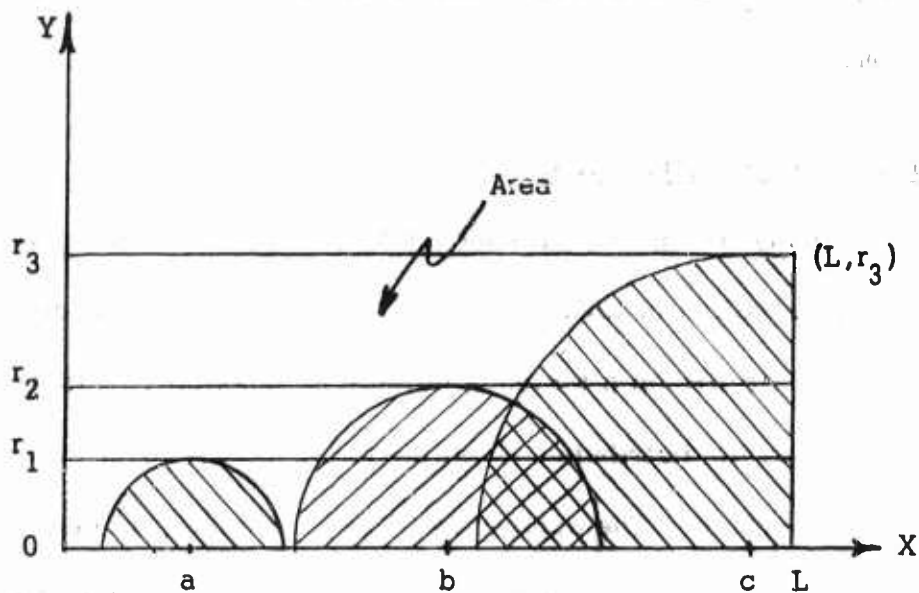


FIGURE 6a

Targets occur in the area according to $f_{X,Y}(x,y)$. X and Y are random variables signifying the coordinates of a target. Ships stationed at a , b , and c can fire on that portion of the area that is shaded.

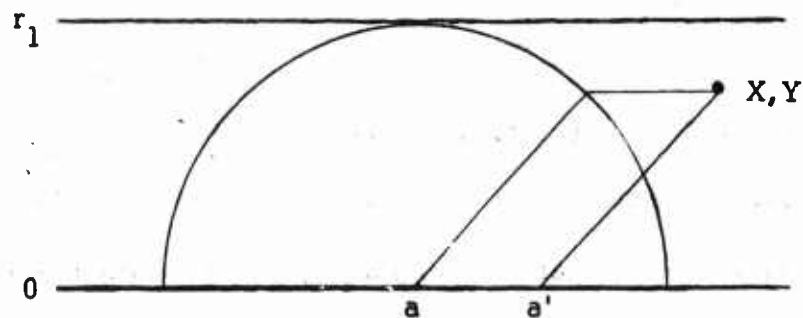


FIGURE 6b

A ship stationed at point a must move to point a' in order to fire on the target at position X,Y .

to $\frac{|a' - a|}{s}$ where s represents the ship's speed (See Figure 6b).

From the figure $r_1^2 = Y^2 + (X - a')^2$

$$a' = X - (r_1^2 - Y^2)^{1/2}$$

or the time required for a ship stationed at point a to reach a position from which a target occurring at X, Y may be fired on is

$$\frac{|X - (r_1^2 - Y^2)^{1/2} - a|}{s}$$

The time required to commence fire once within range is assumed to be negligible compared to the time required to move the ship. In the same fashion the times required for ships stationed at points b and c are

$$\frac{|X - (r_2^2 - Y^2)^{1/2} - b|}{s} \quad \text{and} \quad \frac{|X - (r_3^2 - Y^2)^{1/2} - c|}{s} \quad \text{respectively.}$$

Any mission occurring in the range band $(0, r_1)$ can be fired on by any of the three types of ships, whereas targets between r_1 and r_2 are within range only of cruisers and battleships. Finally targets between r_2 and r_3 are within reach of the battleship only. Thus, the function $g(X, Y, a_1, b_1, c_1)$ becomes

$$\begin{aligned} g_1(X, Y, a, b, c) &= \frac{1}{s} \min \begin{cases} |X - (r_1^2 - Y^2)^{1/2} - a| \\ |X - (r_2^2 - Y^2)^{1/2} - b| \\ |X - (r_3^2 - Y^2)^{1/2} - c| \end{cases} \\ g_2(X, Y, b, c) &= \frac{1}{s} \min \begin{cases} |X - (r_2^2 - Y^2)^{1/2} - b| \\ |X - (r_3^2 - Y^2)^{1/2} - c| \end{cases} \\ g_3(X, Y, c) &= \frac{|X - (r_3^2 - Y^2)^{1/2} - c|}{s} \end{aligned}$$

Therefore, the expected time late may be represented as

$$E[g(X,Y,a,b,c)] = \int_0^L \int_0^{r_1} g_1(X,Y,a,b,c) f_{X,Y}(x,y) dx dy + \\ \int_0^L \int_0^{r_2} g_2(X,Y,b,c) f_{X,Y}(x,y) dx dy + \\ \int_0^L \int_0^{r_3} g_3(X,Y,c) f_{X,Y}(x,y) dx dy = h(a,b,c)$$

Numerous methods are available for obtaining the minimum to $h(a,b,c)$ and no particular method is suggested here. The classical procedure would be to take the appropriate partial derivatives $(\frac{\partial h}{\partial a}, \frac{\partial h}{\partial b}, \frac{\partial h}{\partial c})$, set each equal to zero and solve the resulting set of simultaneous equations. The second order conditions must be checked to assure the proper extreme is obtained. The decision rule is to select that combination of naval gun systems (ships) that provides the minimum expected time late for a specified budget. This technique, although conceptually straightforward, becomes exceedingly cumbersome for hand computations even with simple distributions and a small weapon system.

IV. COST

The cost of an alternative is an integral part of cost-effectiveness analysis. As has been previously mentioned, cost represents expenditure of limited resources. If a greater cost is incurred than necessary to accomplish an objective, it is conceivable that somewhere there exists an objective for which insufficient resources will be available. The object of this chapter, then, is to develop a means of determining the cost of naval gun systems.

Relevance of Historical Cost

The cost of a naval gun system is not always composed of the cost of the research and development, investment, and operating cost. For example, the research and development cost of an existing system is not considered in comparing alternatives. The money has been spent and is presumably irretrievable. Only expenditures that occur pursuant to a choice of a particular alternative are considered. In other words, sunk or historical cost should not be included.

The notion of sunk cost has been extensively treated in the accounting and economic literature, notably in Reference 5, and is basic to analysis designed to distinguish between alternatives. Further discussion of the theoretical aspects concerning exclusion of sunk cost will be omitted.

Categories of Costs

A summary of cost categories which should be reviewed to select pertinent costs may be found in Reference 10. An example of a cost model for the total cost of a naval gun system is shown in Figure 7. Of the items listed only a few are incurred as a direct result of the decision to employ a ship in a naval gunfire role. Remaining costs are independent of such a decision; that is, they are incurred regardless of a decision concerning employment. Selecting the appropriate categories is only part of the solution and several aspects of each category must be considered in more depth.

It is possible that there may be more than one cost applicable to ammunition. If it were decided that current stocks of ammunition would be expended without replacement, the proper cost would be either zero, assuming no opportunity cost for existing stocks, or the cost of overhauling overage units prior to issue and use, whichever was applicable. However, if expended ammunition is to be replaced, the cost would be the overhaul cost (if applicable) and the cost of newly manufactured replacement ammunition. Either method of costing ammunition is appropriate depending on the circumstances. The latter method is probably the most likely.

Operating costs should be included if particular naval gun systems represent additions to the operating forces as a result of activation of mothballed units. This operating cost would be reduced by subtracting

INVESTMENT

a. SCN Cost per Ship

1. New Construction

2. Conversion

b. OPN

1. Fram II

2. Alternation per Ship

3. Expendable Ordnance

c. S&F Cost

1. Fram II Equipment Investment per Ship

2. Fram II Rehabilitation per Ship

3. Alternation Installation per Overhaul

d. PAMN Missiles

OPERATING

I. Direct Costs

a. Ship Personnel Pay and Allowances

b. Medical Care

c. O&M Ships and Facilities

1. Regular Overhaul

2. Non-Scheduled Repair

3. Supplies and Equipage

4. Fuel and Utilities

d. OPN-SSE Maintenance Material

e. OPN Expendable Ordnance

f. PAMN Missiles

II. Indirect Costs

a. Military Personnel Pay and Allowances for Training and Other Support Personnel

FIGURE 7

housekeeping cost of mothballed ships . The operating costs of ships currently in the operating forces will be incurred regardless of a decision concerning employment and can be disregarded in this analysis .

Activation costs are of course peculiar to ships which must be drawn from the reserve fleet and are differential costs due to the decision to employ mothballed ships in a naval gunfire support role .

Finally, the opportunity cost must be considered . In the neo-classical economic literature opportunity cost is defined as the maximum amount which a good or service could yield if applied to some other purpose . It has been argued (Ref. 13) that in the event a viable alternative mission exists, i.e., a mission which would be undertaken if the naval gunfire mission were cancelled, the costs of performing the alternative mission will substitute for the benefits which would be realized . Implicit in this statement is the assumption that costs and benefits are commensurable . These same benefits are, then, the opportunity cost of pursuing the naval gunfire mission to the exclusion of the alternate mission . This argument can be extended to show that the costs of the alternate mission represent only the lower bound on the benefits which would accrue from the alternate mission . Consider some of the alternative missions which may occur .

1. Coastal surveillance and junk patrol
2. Search and rescue
3. Escort
4. Port visit

To show that at least a minimum is represented is straightforward. If the benefits of a mission did not at least equal the costs, the mission would not be undertaken. This, of course, assumes the decision maker is behaving in an optimal fashion. Demonstrating that the costs may provide only a lower bound requires a closer examination. The costs of a mission such as search and rescue would probably be closely approximated by the operating cost if it is assumed the ship would be engaged in a port visit were it not for the search and rescue mission. In this case, operating cost refers to the cost of expendibles used by a ship underway, fuel, etc. Since the alternative is to do nothing, operating cost is generally used in a broader sense, including such items as pay and allowances for the crew; however, the usage here is more restricted. So assuming that the operating costs approximate the cost of a mission, what are the benefits? The benefits of a search and rescue mission are the saving of lives. The costs and benefits are not commensurable; however, the value of a human life is certainly worth the price of expendibles used in steaming and probably a lot more. For example, the value of a human life is presumably worth at least the amount it costs to train a replacement and probably a lot more. Similar arguments can be employed for the other alternate missions with the outcome of each attempting to relate operating cost to human life. Thus, there seems to be no completely satisfactory simple means of determining the opportunity cost of missions alternate to naval gunfire missions.

A possible answer to this dilemma might be to employ a mathematical programming model to allocate ships to the various types of missions for which they are suited using the dual variables, sometimes called the internal prices, as estimates of the opportunity cost or the worth of the next ship in a particular employment*. Reference 9 provides one method of dealing with this particular area of incommensurability (human life).

Therefore, the weapon system cost is composed of the cost of the ammunition, operating cost (if applicable), activation cost (if applicable), and opportunity cost. Specifically, the operating cost might be the daily cost times the number of days the weapon system will be engaging the specified threat. The activation cost might be represented by

$$\frac{\text{no. of days system will engage specified threat}}{\text{estimated total days in remaining lifetime}} \times \text{activation cost}$$

prorating the activation cost over the remaining lifetime. This technique is applicable only to studies that occur before the fact (activation). Once activation is initiated the cost becomes a sunk cost and is omitted from further differential cost analysis.

Assuming successful resolution of the opportunity cost problem, the opportunity cost would be given by the number of missions existing in the specified threat times the maximum opportunity cost from among alternate missions.

*Subject to the problems associated with these variables in integer programming problems.

ERRATA

- Page 7, lines 5/6: hyphenate "histori-cally" vice
"historic-ally"
- Page 12, line 21: "provide" vice "provides"
- Page 20, Fig. 5, title: "Dimensions" vice "Demensions"
- Page 27, line 7: "vertical" vice "verticle"
- Page 27, lines 12/13: hyphenate "simulta-neously" vice
"simultan-iously"
- Page 31, lines 5/6: hyphenate "de-stroyer" vice "es-troyer"
- Page 41, items b2 and c3: "Alteration" vice "Alternation"
- Page 43, lines 9 & 17: "expendables" vice "expendibles"
- Page 50, line 9: "war" vice "was"
- Page 58, line 1: "complementary" vice "complimentary"

V. COST EFFECTIVENESS

Cost effectiveness was previously defined as the systematic analysis of the cost and effectiveness of alternate methods of accomplishing an objective. Of course, what is meant is that costs and effectiveness are to be simultaneously compared and that comparison is to be the subject of this chapter.

In a graph of effectiveness as a function of cost it classically assumes the form shown in Figure 8.

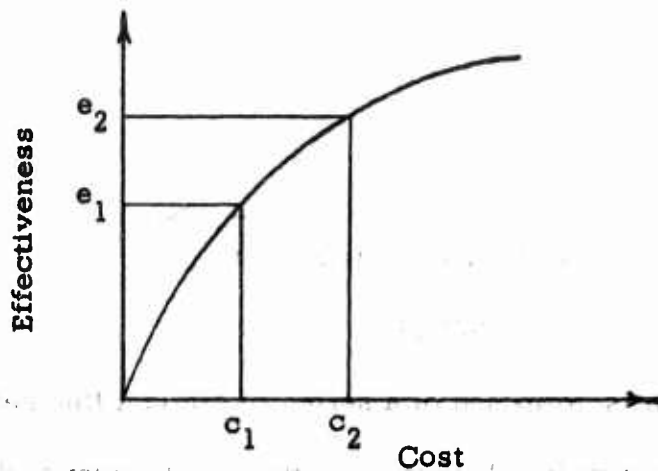


FIGURE 8

With the aid of analysis users are able to determine the effectiveness of a specific budget expenditure or the cost necessary to provide a particular level of effectiveness. Furthermore, repeating a portion of the argument in Chapter I, it is possible to express the additional cost necessary, $c_2 - c_1$, to increase the effectiveness from e_1 to e_2 . As the curve indicates, the law of diminishing marginal returns* applies as it becomes increasingly costly to improve the effectiveness

*It is not entirely clear that this does in fact represent diminishing marginal returns as opposed to diminishing returns to scale.

by some increment Δe . Now the behavior of effectiveness vs. cost in the specific situations considered herein must be examined.

Bombardment Scenario

It can be shown that a curve of the number of times (K) a threat can be defeated will have the general behavior shown in Figure (see Appendix B).

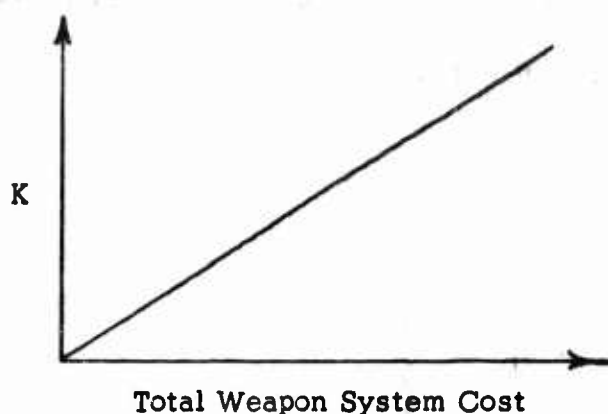


FIGURE 9

Actually, the curve is composed of a series of straight line segments; however, the macroscopic behavior is as shown. In order to develop the curve it is necessary to determine (K) for various budget levels. It should be noted that a specified budget level can result in more than one value of (K). In fact, it may happen that one value of (K) can be found for each possible combination of naval gun systems that the budget level is capable of funding. For example, if the budget level were 10 units and naval gun systems cost 1, 2, and 3 units, respectively, it would be possible to fund any one of the following combinations of naval gun systems:

10 of system 1 or
5 of system 2 or
3 of system 3 or
1 of system 1, etc.

Some combination(s) of naval gun systems will yield a maximum value for (K) . That this is so may be illustrated by considering the combination of 10 of system 1. If system 1 was the system having a maximum range r_1 , it would not be possible for system 1 to obtain even a (K) of 1, for some targets would be out of range. One aspect of the behavior of effectiveness vs. cost requires some additional consideration. Absent is the diminishing marginal returns behavior illustrated in Figure 8. If, in fact, the ability to defeat the threat on the 20-30th confrontation is just as important as the 0-10th time, the law of diminishing marginal returns does not apply. In truth, however, the ability to defeat the threat a great number of times is probably not interesting. In this case, a curve of effectiveness versus K would probably behave as shown in Figure 10, thus restoring the notion of diminishing marginal returns.

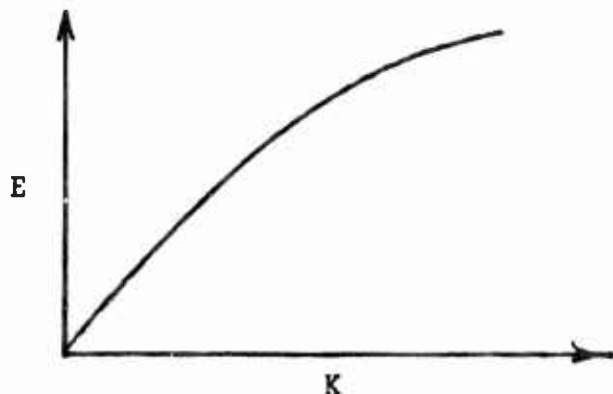


FIGURE 10

Call Fire Scenario

One method of displaying graphically the results of the call fire scenario model is shown in Figure 13 (see page 50). As in the preceding model, a specified budget level can yield more than one combination of naval gun systems; thus, several values for expected time late may result. The curve is developed from the least of these values, in this context synonymous with most effective. Varying the budget level will provide a profile of expected time late.

Caution should be exercised when using a criterion involving expected value. The behavior of a random variable about its mean (μ) can vary considerably depending upon the underlying distribution. For example, in Figure 11, the case of a normal distribution with a small variance, the behavior is generally good and the expected value is a reasonable value to use. This is because most values that occur are near the mean.

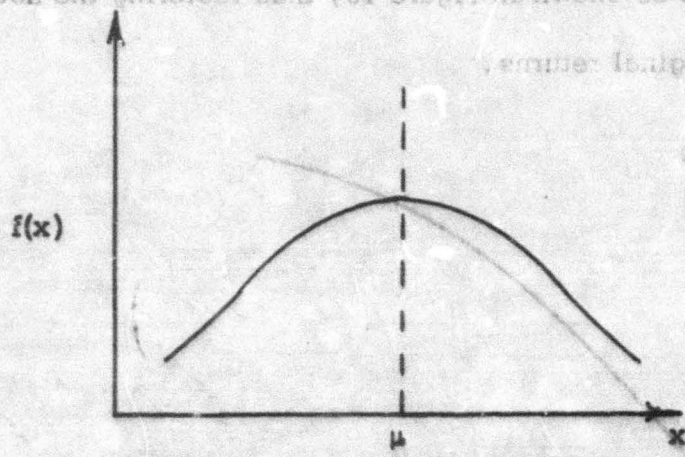


FIGURE 11

However, in the case of a bimodal distribution, as shown in Figure 12, the expected value is of little value and should be avoided.

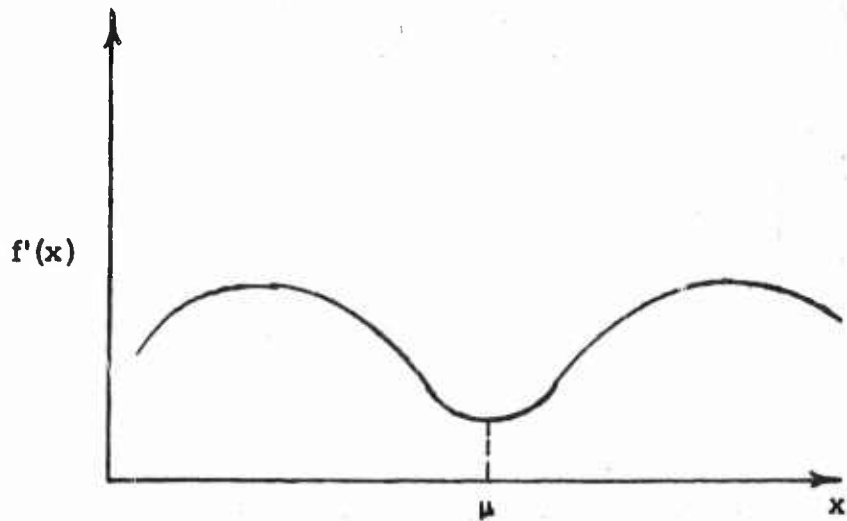


FIGURE 12

If the number of targets is assumed to be large compared to the number of ships, the distribution of time late may be approximated by the normal distribution (Ref. 12, p. 238). By computing the variance of time late, a confidence interval, shown by the dotted lines, can be constructed about the expected value, thus providing subjective confidence regarding the upper and lower limits of the time late which can be expected for a particular weapon system.

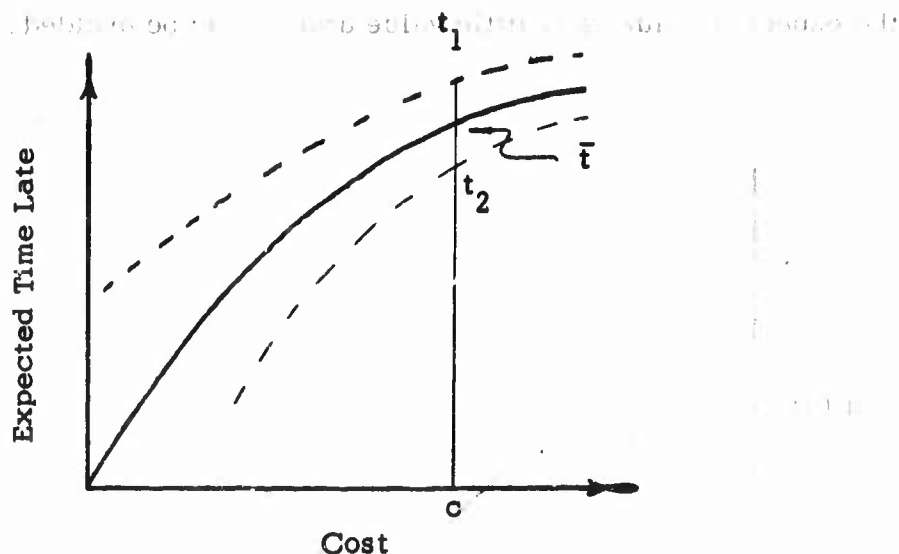


FIGURE 13

The convergent behavior may be substantiated by letting t_1 = time late for replication (i), ($i = 1, 2, \dots, n$) of a particular weapon system confronting a specified threat. Then \bar{t} , the mean of the random sample, is normally distributed with mean μ and variance σ^2/n . As $n \rightarrow \infty$, $\sigma^2/n \rightarrow 0$. Thus, the confidence region will exhibit the converging behavior shown (Ref. 8, p. 226).

As was mentioned, the foregoing presents a methodological approach that has general application to naval gunfire. Although the approach here was undoubtedly influenced by the current war in Vietnam, effort was made to remain sufficiently general to preclude early obsolescence of the procedures developed. One interesting area of application of this work would be the determination of an optimal weapon suit for a new class ship designed primarily to provide gunfire support. An additional area worthy of further effort is the determination

of a suitable allocation model for combat forces and one that would,

as a side benefit, yield the opportunity costs mentioned. Finally,

the problem of incommensurability of costs and benefits (value of

human life) is a continuing area of difficulty and always worthy of

additional attention.

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(13) Steiner, P. O. "The Role of Alternative Cost in Project Design and Selection". *Quarterly Journal of Economics*, LXXIX (August, 1965), pp. 417-429.

(14) Supporting Arms in Amphibious Operations. NWIP SS-2(A). A procedure that can be employed to determine the effect of variations in the draft of different ships is shown in the figure and accompanying explanation.

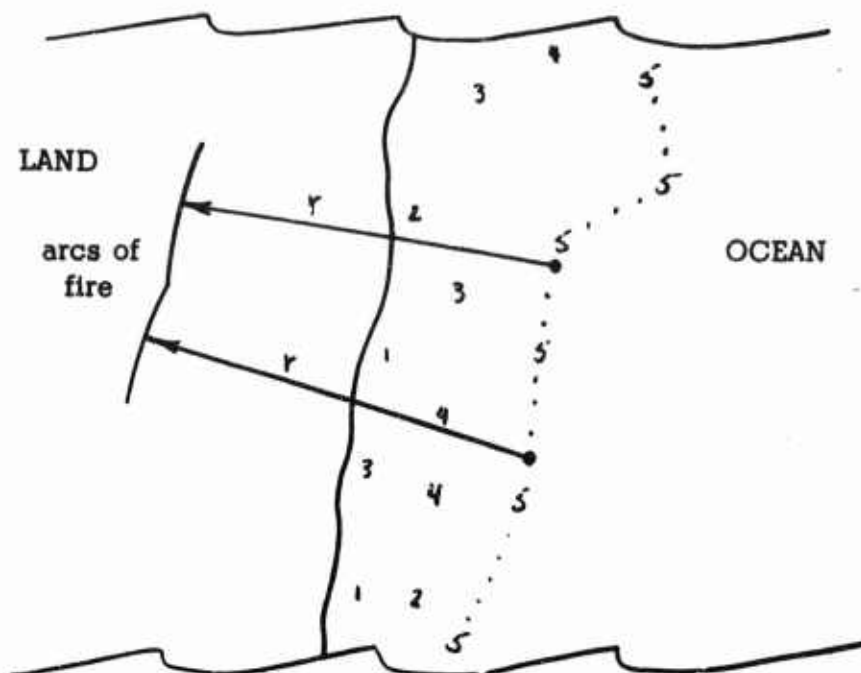


FIGURE A-1

The figures 1, 3, 5, etc., are found on hydrographic charts and represent the depth of the water in fathoms. Determine the limiting draft (the shallowest water into which a ship can safely proceed) for the particular ship under consideration and connect the figures signifying that limiting draft, e.g., 5 fathoms. Strike a series of arcs from

this limiting draft line with a radius equal to the range of the gun being considered. By connecting the arcs which extend the farthest from the coast, an envelope of the land within range of the naval gun can be determined. Of course, different ships have different gun ranges and limiting drafts. By repeating this procedure for different ships, a comparison of the portion of the countryside within range of the guns of various ships was made. It was discovered that in only two locations could a destroyer fire further inland than a cruiser or battleship because of a shallower draft. Thus, draft was determined not to be of significance.

APPENDIX B

It has been asserted that K behaves linearly as the size of weapon systems is increased. This appendix will be a proof of this assertion. Assume that $m = s = 2$, the program can then be written

$$\min \frac{r_{11}}{n_1} x_{11} + \frac{r_{12}}{n_1} x_{12} + \frac{1}{n_1} L_1 \quad (1)$$

subject to

$$\begin{aligned} n_1[r_{21} x_{21} + r_{22} x_{22} + L_2] &= n_2[r_{11} x_{11} + r_{12} x_{12} + L_1] \\ x_{11} + x_{21} &= f_1 \end{aligned} \quad (2)$$

$$x_{21} + x_{22} = f_2$$

$$\text{and } x_{ij} \geq 0, L_j \geq 0 \quad \text{for all } i, j$$

This can also be written

$$\min \frac{r_{11}}{n_1} x_{11} + \frac{r_{12}}{n_1} x_{12} + \frac{1}{n_1} L_1 \quad (3)$$

subject to

$$\begin{aligned} n_2 r_{11} x_{11} + n_2 r_{12} x_{12} + n_2 L_1 - n_1 r_{21} x_{21} - n_1 r_{22} x_{22} - n_1 L_2 &= 0 \\ x_{11} &+ x_{21} &= f_1 \\ x_{12} &+ x_{22} &= f_2 \end{aligned} \quad (4)$$

$$\text{and } x_{ij} \geq 0, L_j \geq 0 \quad \text{for all } i \text{ and } j$$

The dual linear program can then be written

$$\max f_1 y_1 + f_2 y_2 \quad (5)$$

$$\begin{aligned} \text{subject to} \quad n_2 r_{11} u + y_1 &\leq r_{11}/n_1 \\ n_2 r_{12} u + y_2 &\leq r_{12}/n_1 \\ n_2 u &\leq 1/n_1 \\ -n_1 r_{21} u + y_1 &\leq 0 \\ -n_1 r_{22} u + y_2 &\leq 0 \\ -n_1 u &\leq 0 \\ u, y_1, y_2 &\text{ unrestricted.} \end{aligned} \quad (6)$$

Equations (5) and (6) result from associating u , y_1 and y_2 with the first, second, and third variables in equation (4), respectively.

Solving equation (6), it can be shown that

$$\begin{aligned} u &\leq 1/n_1 n_2 \\ u &\geq 0 \\ \therefore 0 &\leq u \leq 1/n_1 n_2 \\ \text{and } y_1 &\leq \min \left[\frac{r_{11}}{n_1} - n_2 r_{11} u, n_1 r_{21} u \right] \\ y_2 &\leq \min \left[\frac{r_{12}}{n_1} - n_2 r_{12} u, n_1 r_{22} u \right] \end{aligned} \quad (7)$$

If $u = 0$, then $y_1 = y_2 = 0$,

or $u = \frac{1}{n_1 n_2}$, then $y_1 = y_2 = 0$.

Therefore, in order that $y_1 > 0$ and $y_2 > 0$, it must hold that $u > 0$.

From this complimentary slackness implies that $L_1 = L_2 = 0$.

Therefore, from the y_1 restraint

$$u = \frac{r_{11}}{n_1(n_1 r_{21} + n_2 r_{11})} \quad (8)$$

and from the y_2 restraint

$$u = \frac{r_{12}}{n_1(n_1 r_{22} + n_2 r_{12})} \quad (9)$$

It is not obvious that (8) and (9) are consistent; nevertheless the primal can be written

$$\min \frac{r_{11}}{n_1} x_{11} + \frac{r_{12}}{n_1} x_{12} + \frac{1}{n_1} L_1 \quad (10)$$

subject to

$$n_2 r_{11} x_{11} + n_2 r_{12} x_{12} + n_2 L_1 - n_1 r_{21} (f_1 - x_{11}) - n_1 r_{22} (f_2 - x_{12}) - n_1 L_2 = 0$$

$$\text{or } (n_2 r_{11} + n_1 r_{21}) x_{11} + (n_2 r_{12} + n_1 r_{22}) x_{12} + n_2 L_1 - n_1 L_2 = n_1 r_{21} f_1 + n_1 r_{22} f_2 \quad (11)$$

Thus, the primal actually has only one restraint and, hence, only one of the variables in (11) can be positive. If $x_{11} = x_{12} = L_1 = 0$, then $L_2 < 0$ which violates the requirement that $L_2 \geq 0$. Thus $L_2 = 0$.

The objective function in the primal can then be written

$$\min \left[\frac{r_{11}(r_{21}f_1 + r_{22}f_2)}{n_2 r_{11} + n_1 r_{21}}, \frac{r_{12}(r_{21}f_1 + r_{22}f_2)}{n_2 r_{12} + n_1 r_{22}}, \frac{r_{21}f_1 + r_{22}f_2}{n_2} \right]$$

which supports the contention that K behaves linearly. Figure B-1 illustrates this behavior. On a microscopic scale K consists of a series of straight line segments.

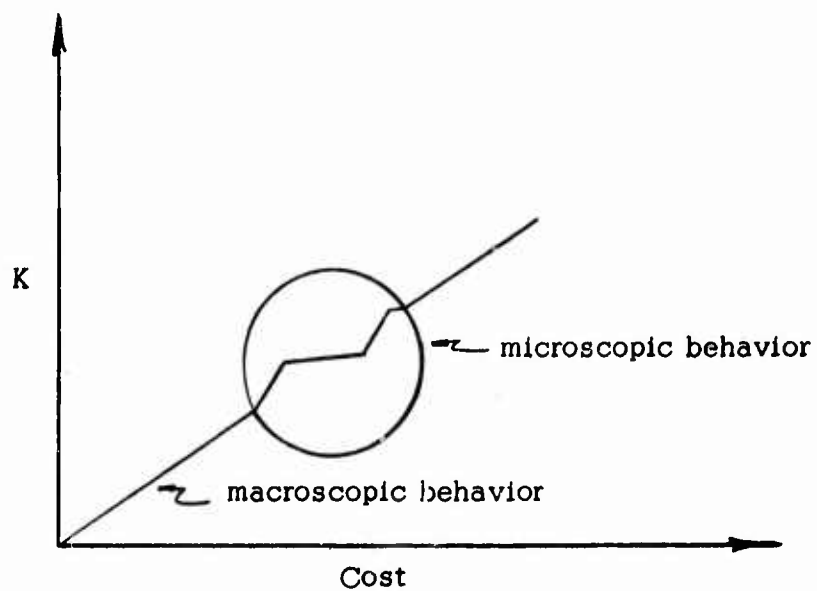


FIGURE B-1

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13. ABSTRACT		
<p>In this thesis a methodological approach to the determination of the cost effectiveness of naval gunfire support is developed. Two models are presented. The first is a linear program developed by the Ballistics Research Laboratory, in which naval gunfire is employed against a relatively static threat. The second is a probabilistic model wherein the capability of naval gun systems against transient targets is treated. Included is a discussion of the theoretical considerations of the cost of naval gun systems which concludes in the presentation of some alternatives of presenting the results of analysis of this type.</p>		

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